Fluctuations in the Universe

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Plan for the lesson

Lesson will be mostly black-board, slides only have crucial equations and points.
Topics covered:

- Mathematical description of the random fields
- Growth of structure in the Universe
- Dark Matter Halos, Halo mode of structure formation

Random fields

- Theories about structure in the universe are theories about the statistics of the structure in the universe.
- Concept of *realization*
- Averages are averages over realizations, not space (though egoticity allows us to switch between the two)
- We will consider isotropic homogeneous *scalar* random fields in N-dimensions
- The basic quantity is an over-density

$$\rho(\mathbf{x}) = \bar{\rho}\,(1 + \delta(\mathbf{x})) \tag{1}$$

with $\delta \in [-1, \infty)$ and $\langle \delta \rangle = 0$

Fourier Transforms

$$\delta_k = \int \delta(x) e^{i\mathbf{k}\cdot\mathbf{x}} d^3 r \quad (2)$$

$$\delta(\mathbf{x}) = \frac{1}{(2\pi)^3} \int \delta_k e^{-i\mathbf{k}\cdot\mathbf{x}} d^3 k \quad (3)$$

Crucial identities:

$$\delta_k^* = \delta_{-k} \text{ for real} \delta \quad (4)$$

$$\int e^{i\mathbf{x}(\mathbf{k}-\mathbf{k}')} d^3 x = (2\pi)^3 \delta^D(\mathbf{k}-\mathbf{k}') \quad (5)$$

$$\nabla \delta(\mathbf{x}) \rightarrow i\mathbf{k}\delta_k \quad (6)$$

$$\delta(\mathbf{x})\eta(\mathbf{x}) = \delta_k * \eta_k \quad (7)$$

Correlators

N-point correlators are basic objects of the type

$$\langle \delta_{k_1} \delta_{k_2} \cdots \delta_{k_N} \rangle \tag{8}$$

- 1-point, mean: $\langle \delta_k \rangle = 0$
- 2-point, power spectrum, *for isotropic and homogeneous fields*

$$\left\langle \delta_k \delta_k' \right\rangle = (2\pi)^3 \delta^D(\mathbf{k} + \mathbf{k}') P(k) \tag{9}$$

- In general, translational invariance always gives you $\delta^D(\sum \mathbf{k}_i)$ prefactor
- For any field, the complete set of correlators gives a complete description
- For normally distributed fields power spectrum is everything (odd correlators vanish, even ones are given by "Wick" expansion)

Correlation function

(2-point) correlation function is the configuration space analog of power spectrum:

$$\left\langle \delta(\mathbf{x})\delta(vx')\right\rangle = \xi(|\mathbf{x}' - \mathbf{x}|) \tag{10}$$

Can show using identities above that

$$\xi(\mathbf{r}) = \frac{1}{(2\pi)^3}\int P(\mathbf{k})e^{-i\mathbf{kr}}d^3k = \frac{1}{2\pi^2}\int P(k)\frac{\sin(kr)}{kr}k^2dk \tag{11}$$

Note that correlations are translationally invariant, but not diagonal. They are dimensionless while power spectrum has units of inverse volume.
They also obey "mass conservation" integral constraint:

$$\int \xi(\mathbf{x})d^3x = 0 \tag{12}$$

Poisson sampling

In cosmology with often deal with individual objects, rather than continous field:

- Probability of finding an object in volume V is $V\bar{n}(1+\delta(\mathbf{x}))$
- Correlation function can be thought of as an excess of pairs wrt to the mean number of pairs at a given separation
- Power spectrum gets a "shot noise" contribution $P_s = \bar{n}^{-1}$

Spherical transforms

We often have to deal with the fact that the sky is spherical. Kinda sucks.

FT equivalent is the Spherical Harmonic Transform. Fourier modes are replace by Spherical harmonics

$$Y_\ell^m(\theta,\phi) \propto P_\ell(\cos\theta)e^{im\phi} \tag{13}$$

- $m = -\ell \cdots \ell$
- Large scales: monopole, dipole, etc.
- Small scales: $\ell \sim |k|$ and $m \sim k_\phi$ and this correspondence is exact for "flat enough" patch in angular units.
- Forward and backwards transform:

$$a_{\ell m} = \int \delta(\theta,\phi) Y_\ell^{m\star}(\theta,\phi) d\Omega \tag{14}$$

$$\delta(\theta,\phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_\ell^m(\theta,\phi) \tag{15}$$

Spherical transforms 2:

Power spectrum equivalent:

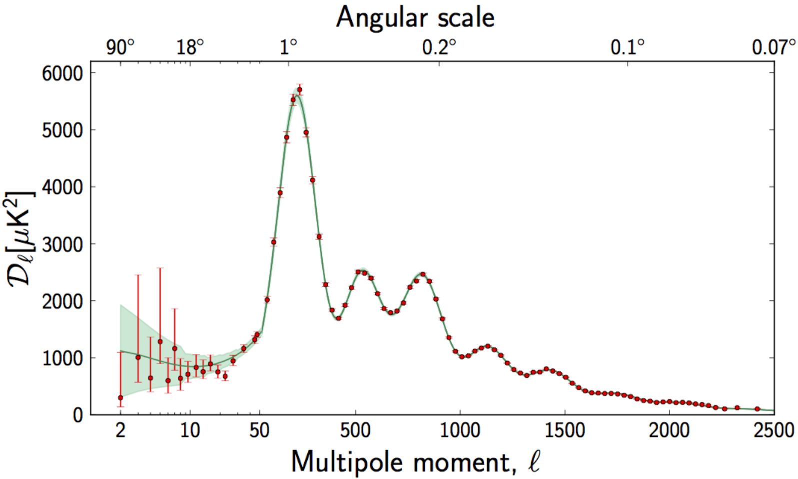
$$C_\ell = \langle a_{\ell m} a^*_{\ell m} \rangle \tag{16}$$

- Rotational invariance leads to no m dependence

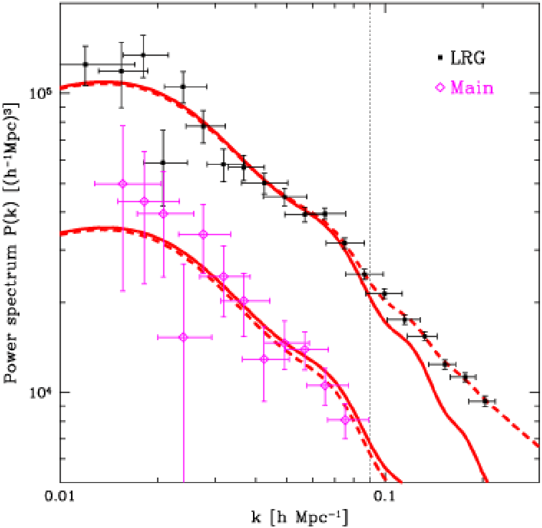
All the intuition from 3D transfers to the sphere as expected.
Projection from 3D to 2D sphere involves:

- A window function
- Triple integral that can be simplified in certain limits
- "Limber approximation" is commonly used

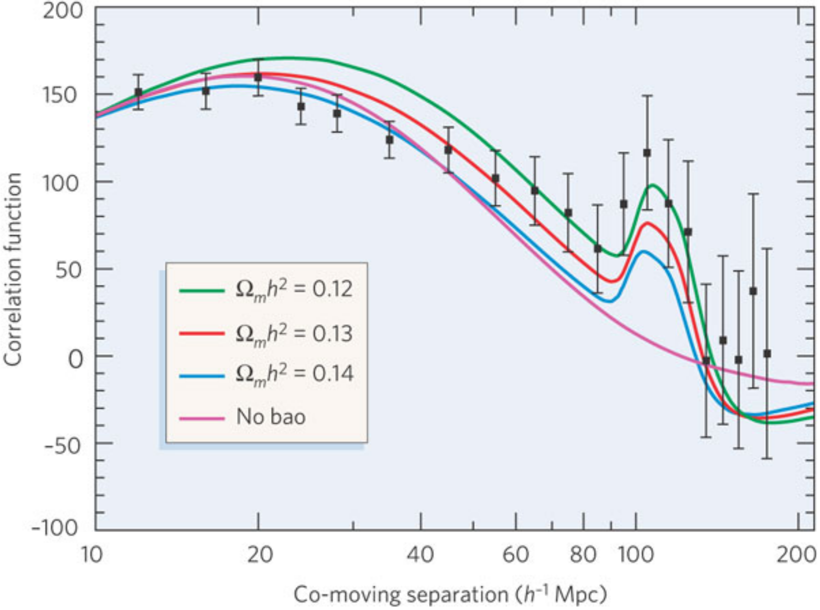
Planck results



Galaxy power spectra

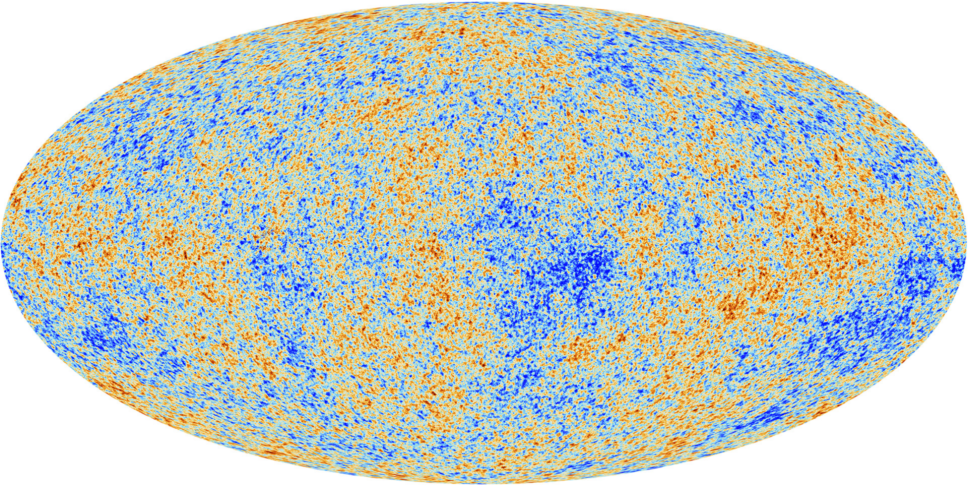


Galaxy correlation function



Evolution of perturbations

- *Very early universe* ($< 10^{-30}$s): inflation generates seed fluctuations. These have flat power spectrum in potential.
- *Pre-recombination universe* $z > 1100$: tightly-coupled photon-baryon plasma, fluctuations linear, each mode develops independently, GR corrections matters
- *Dark ages:* $50 < z < 1100$: passive linear evolution dominated by dark matter fluctuations growth
- *Non-linear universe:* $z < 6$: small scales become non-linear and form rich structures



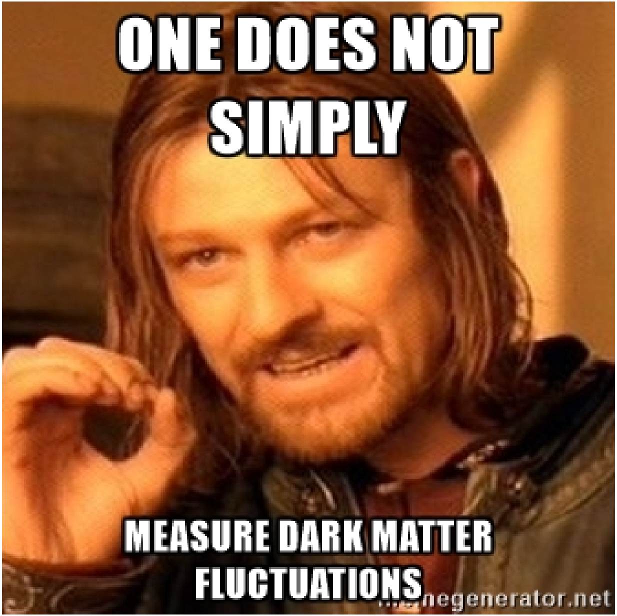
Linear vs non-Linear

- During linear evolution, each Fourier mode grows independently
- After recombination, they grow in a scale-independent fashion

$$P_L(k, z) = \left(\frac{g(z)}{g(z_0)} \right)^2 P_L(k, z_0) \tag{17}$$

- For sufficiently large scales this is still true today
- Small scales form dark-matter halos.

MOVIES


ONE DOES NOT
SIMPLY
MEASURE DARK MATTER
FLUCTUATIONS


WHAT IF I TOLD YOU
GALAXIES TRACE DARK MATTER

Dark Matter Halos

- Non-linear universe strictly non-amenable to exact analytic treatment
- Approximations are either analytic, semi-analytic or pure numerics
- Basic picture is the following:
  - Halos form in peaks in the primordial density fluctuations
  - They follow the linear matter field
  - They acrete baryons that cool and form stars, etc.
  - Galaxies form in these halos (in a prescription of mass)
- Halos has a universal density profile know as Navarro, Frenk and White:

$$\rho(r) = \frac{\rho_0}{\frac{r}{r_s}\left(1 + \left(\frac{r}{r_s}\right)^2\right)} \tag{18}$$

Dark matter from galaxy clustering

Two very robust assumption about the galaxy formation process:

- The only field that matters on large scales are the fluctuations in the matter fluctuations $\rho_m = \bar{\rho}_m(1 + \delta_m)$
- The galaxy formation process is local on some scale R:

$$\delta_g(\mathbf{x}) = F[\delta_m],$$

  where F is an arbitrary functional that, however has no contributions for distances larger than R from $\mathbf{x}$.

Under these assumptions, in the $k \to 0$ limit, galaxies in redshift-space must trace dark-matter following

$$\delta_g(\mathbf{k}) = (b_\delta + b_\eta f \mu^2)\delta_m(\mathbf{k}) + \epsilon,$$

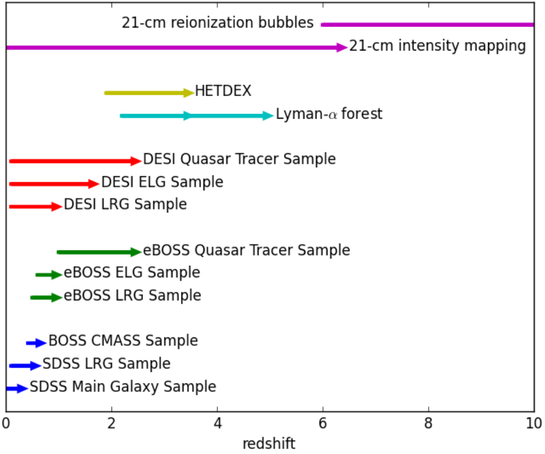
where bs are bias parameters and ϵ is a white noise stochastic variable.

Galaxy clustering: how to do it

So, what we need to do:

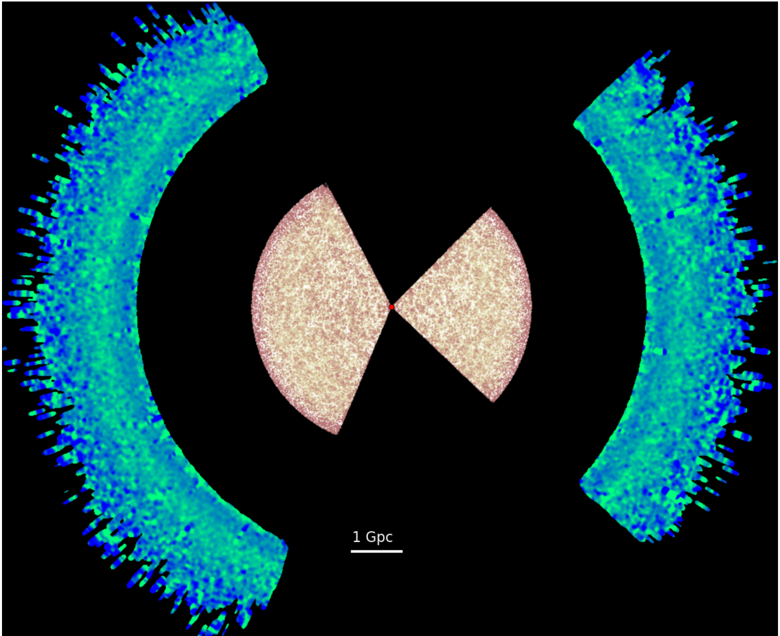
- Take spectra of many galaxies
- Measure redshifts of those galaxies
- Make a map of the universe using those galaxies
- One large scale the resulting map will be the same as map of underlying fluctuations up to a factor
- Our theory for underlying dark matter fluctuations is well understood, even though we do not understand astrophysics very well

From tracers to dark matter

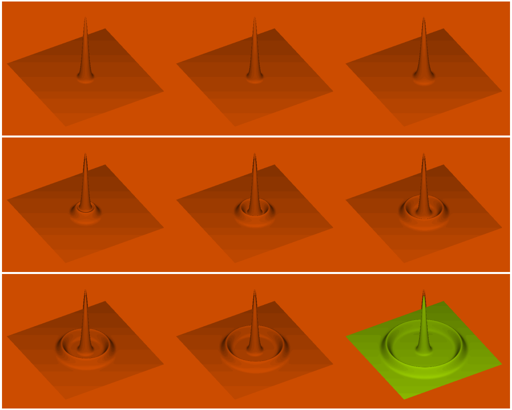


- Disclaimer: plot does not show number densities and does not include photometric experiments
- At $z < 2$ galaxies are best tracers: more galaxies $\rightarrow$ more dark matter
- At $z > 2$ different techniques must be used: Lyman-α forest is one such probe used by BOSS
- Systematics very different between tracers – multiple fundamentally different tracers always useful

BOSS maps

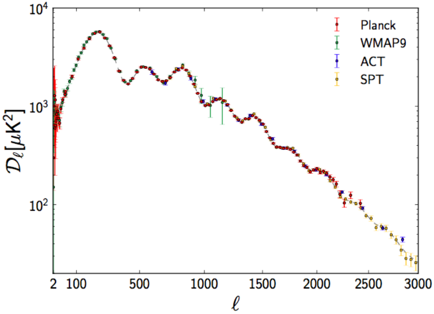


What is BAO?

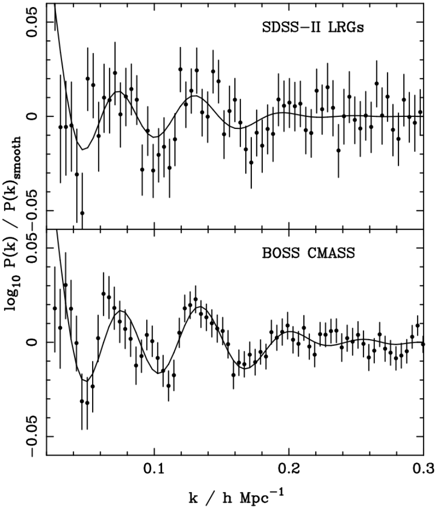


- Before recombination (i.e. formation of hydrogen atoms), primordial plasma supports acoustic waves
- Sound waves travel through Universe as long as it is in primordial plasma state
- We can see them in CMB power spectrum
- The characteristic scale is imprinted as a small bump into the correlation properties of dark matter
- It acts as a standard ruler, allowing very robust measurements of the expansion history of the universe.

What is BAO?

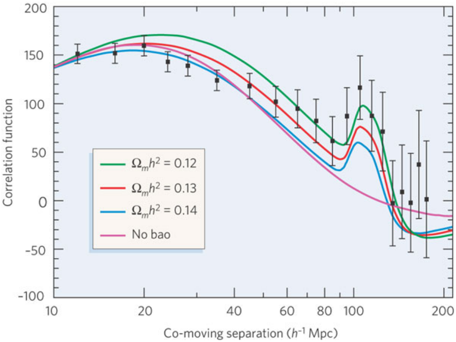


BAO in Cosmic Microwave Background

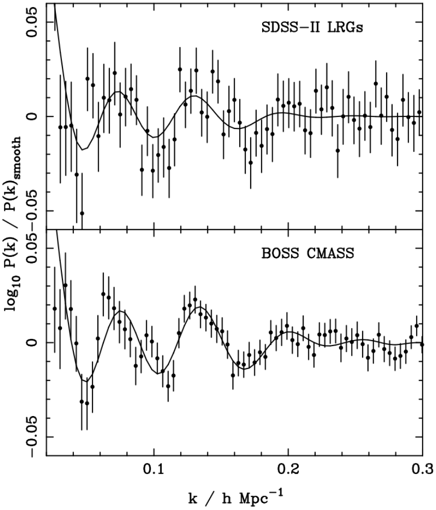


BAO in CMASS galaxies

What is BAO?

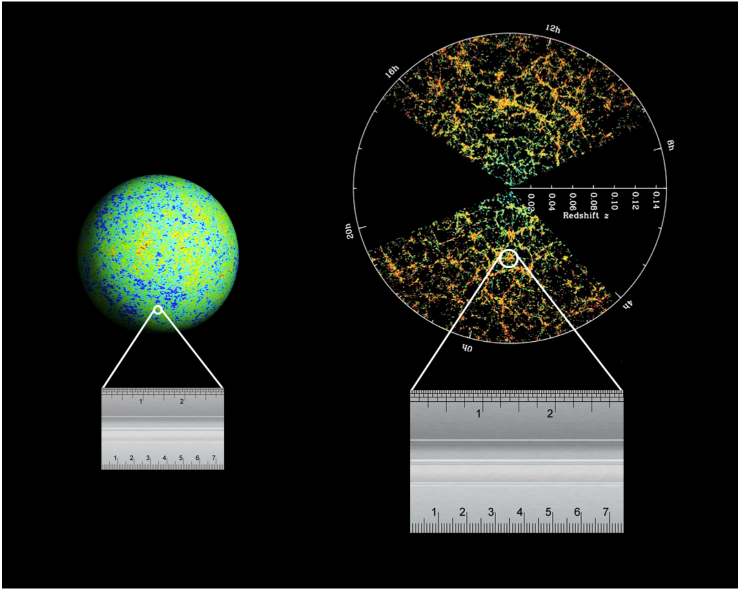


BAO in correlation function

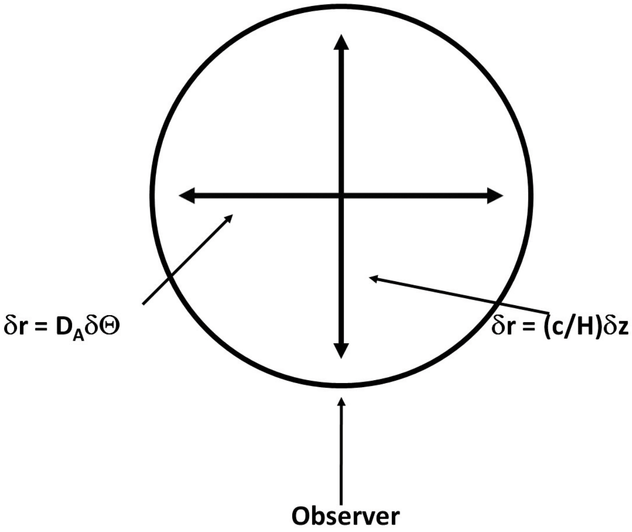


BAO in power spectrum

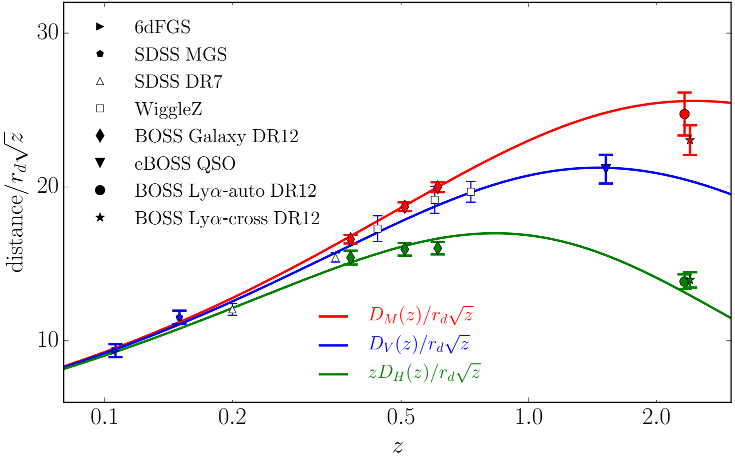
BAO is a statistical ruler



BAO is a statistical ruler



BAO distance measurement



Other important missing pieces

- Redhift-space distortions
- photometric vs spetroscopic survey
- gravitational lensing